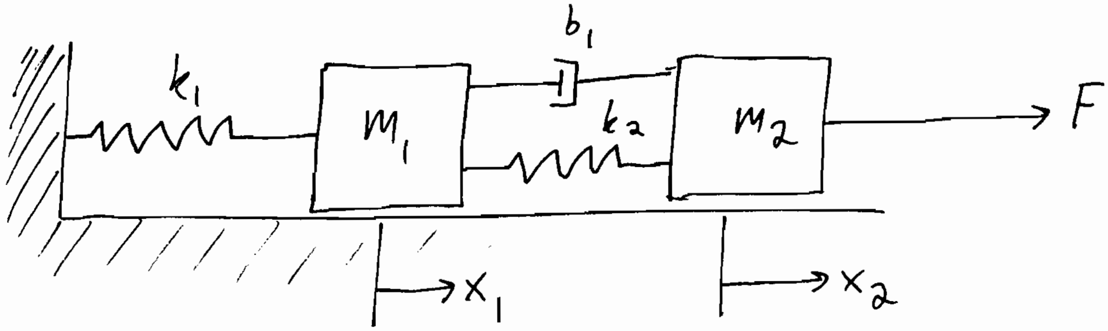
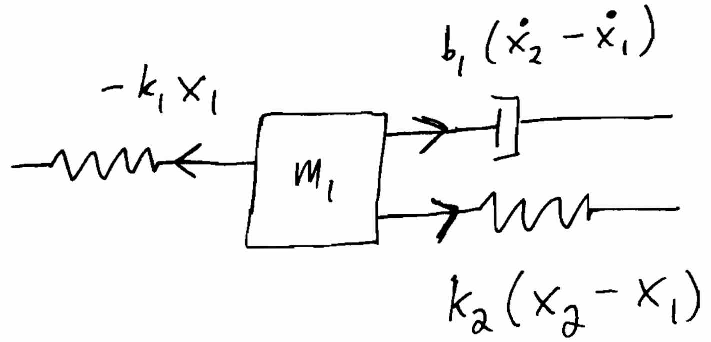
#1).



For m_1 :

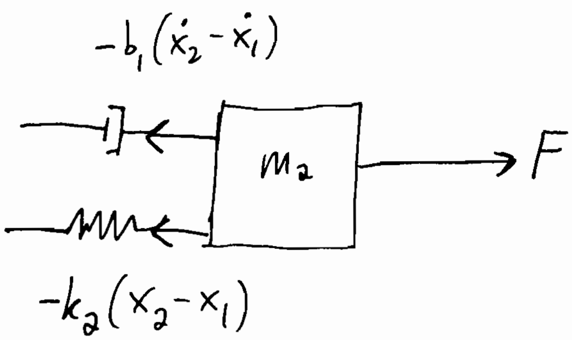


So $m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2-x_1) + b_1(\dot{x}_2-\dot{x}_1)$

or $m_1\ddot{x}_1 = -k_1x_1 - k_2(x_1-x_2) - b_1(\dot{x}_1-\dot{x}_2)$

or $m_1\ddot{x}_1 + k_1x_1 + k_2(x_1-x_2) + b_1(\dot{x}_1-\dot{x}_2) = 0$

For m_2:



$$m_2\ddot{x}_2 = F - k_2(x_2-x_1) - b_1(\dot{x}_2-\dot{x}_1)$$

or $m_2\ddot{x}_2 = F + k_2(x_1-x_2) + b_1(\dot{x}_1-\dot{x}_2)$

or $m_2\ddot{x}_2 + k_2(x_2-x_1) + b_1(\dot{x}_2-\dot{x}_1) = F$

or $m_2\ddot{x}_2 - k_2(x_1-x_2) - b_1(\dot{x}_1-\dot{x}_2) = F$

Problem 2 $\quad y(t) = e^t x(t)$

Homogeneous: Consider $\bar{x}(t) = A\,x(t)$.

Then $\bar{y}(t) = e^t \bar{x}(t) = e^t \cdot A\,x(t) = A\,y(t)$.

Time variant: Consider $\bar{x}(t) = x(t-t_0)$. Then:

$\bar{y}(t) = e^t \bar{x}(t) = e^t x(t-t_0) \neq y(t-t_0)$

$\left(y(t-t_0) = e^{t-t_0} x(t-t_0)\right)$

$\Rightarrow$ not time invariant, so it is time varying

BIBO Unstable: Consider a bounded input,

$x(t) = u(t)$. For $t \geq 0$, $u(t) = 1$.

$\Rightarrow y(t) = e^t u(t) = e^t$ for $t \geq 0$.

This is not bounded as $t \to \infty$.

Problem 3, iii

$$h(t) = e^{\frac{-1-j}{2}t}\,u(t) - e^{\frac{-1+j}{2}t}\,u(t)$$

$$x(t) = 7\,u(t-1)$$

$$y(t) = h(t) * x(t) = \underbrace{\int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau}_{\text{Pick this one}} = \int_{-\infty}^{\infty} h(t-\tau)\,x(\tau)\,d\tau$$

Let $\lambda_1 = \frac{-1-j}{2}$ and $\lambda_2 = \frac{-1+j}{2}$.

$$y(t) = \int_{-\infty}^{\infty} \left[e^{\lambda_1 \tau} u(\tau) - e^{\lambda_2 \tau} u(\tau)\right] \cdot 7\,u(t-\tau-1)\,d\tau$$

$$= \left[\int_{-\infty}^{t-1} \left[e^{\lambda_1 \tau} - e^{\lambda_2 \tau}\right] u(\tau) \cdot 7\,d\tau\right] \cdot u(t-1) \leftarrow \text{(otherwise bounds of integration don't make sense)}$$

$$= \left(\int_{0}^{t-1} \left[e^{\lambda_1 \tau} - e^{\lambda_2 \tau}\right] \cdot 7\,d\tau\right) \cdot u(t-1)$$

$$= \left(7\left[\frac{1}{\lambda_1} e^{\lambda_1 \tau} - \frac{1}{\lambda_2} e^{\lambda_2 \tau}\right]_{\tau=0}^{t-1}\right) \cdot u(t-1)$$

$$= 7\left[\frac{1}{\lambda_1}\left(e^{\lambda_1 (t-1)} - 1\right) - \frac{1}{\lambda_2}\left(e^{\lambda_2 (t-1)} - 1\right)\right] u(t-1)$$

$$\boxed{y(t) = \left[\frac{14}{-1-j}\left(e^{\frac{-1-j}{2}(t-1)} - 1\right) - \frac{14}{-1+j}\left(e^{\frac{-1+j}{2}(t-1)} - 1\right)\right] u(t-1)}$$

$$J_1\ddot{\theta} + b_1\dot{\theta} + \theta = K(V_a - K_e\dot{\theta})$$

$$\ddot{\theta} + \frac{b_1}{J_1}\dot{\theta} + \frac{1}{J_1}\theta = \frac{K}{J_1}V_a - \frac{K_e}{J_1}\dot{\theta}$$

$$\ddot{\theta} = -\frac{(KK_e+b_1)}{J_1}\dot{\theta} - \frac{1}{J_1}\theta + \frac{K}{J_1}V_a$$

let the state vector be $\begin{pmatrix}\theta\\ \dot{\theta}\end{pmatrix} = \vec{x}$

$\therefore\ \dot{\vec{x}} = \begin{pmatrix}\dot{\theta}\\ \ddot{\theta}\end{pmatrix}$ and we can re-write

$$\cancel{\begin{pmatrix}\dot{\theta}\\ \ddot{\theta}\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\frac{(KK_e+b_1)}{J_1} & -\frac{(K_e+b_1)}{J_1}\end{pmatrix}\begin{pmatrix}\theta\\ \dot{\theta}\end{pmatrix}}$$

$$\begin{pmatrix}\dot{\theta}\\ \ddot{\theta}\end{pmatrix} = \begin{pmatrix}0 & 1\\ -\frac{1}{J_1} & -\frac{(KK_e+b_1)}{J_1}\end{pmatrix}\begin{pmatrix}\theta\\ \dot{\theta}\end{pmatrix} + \begin{pmatrix}0\\ \frac{K}{J_1}\end{pmatrix}V_a$$

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consider $\lambda_{1,2} = \frac{-1 \mp \sqrt{1-J_1}}{J_1}$

stability occurs when $Re\{\lambda\} < 0$

if $0 < J_1 \leq 1 \Rightarrow \sqrt{1-J_1} \geq 0$ and $\sqrt{1-J_1} < 1$

$\therefore\ \lambda_{1,2} = \frac{-1 \mp x}{J_1}$ where $x = \sqrt{1-J_1}$ and $0 \leq x < 1$

so $\lambda_{1,2} < 0$ (and not complex) $\Rightarrow$ stable

if $J_1 > 1$: $\sqrt{1-J_1} = j\sqrt{J_1-1}$

and $\lambda_{1,2} = \frac{-1}{J_1} \mp j\frac{\sqrt{J_1-1}}{J_1}$ and $Re\{\lambda_{1,2}\} = -\frac{1}{J_1} < 0$

again stable