Signals and Systems - MAE 143A - Quiz 2, 2009

Student name and number _____

NOTE: For all the questions you need to REASON/EXPLAIN your answer.

1. (5 points) Find an ODE model for the mechanical system shown in Figure 1.

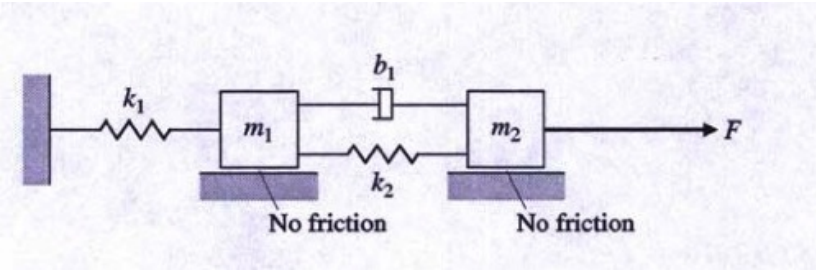

Figure 1: Multibody system for Problem 1

2. (3 points) A system is excited by an input signal $x(t)$ and returns the output signal:

$$y(t) = |\cos(2\pi\omega_c t)|(x(t) + \alpha),$$

where $\omega_c > 0$ and $\alpha > 0$ are certain real constants. Show that the system is not homogeneous, but it is memoryless and BIBO stable.

3. (7 points) The rotational dynamics of a motor is described by the ODE:

$$J_1\ddot{\theta} + b_1\dot{\theta} + \theta = K(V_a - K_e\dot{\theta}),$$

where J_1, b_1, K, and K_e, are motor constants, θ describes the angle of the motor load, and V_a is an applied input voltage. Do the following:

   (i) Determine the system state and a state-space representation of the system.

   (ii) When $b_1 + KK_c = 2$, the associated impulse response of this system is $h(t) = \frac{K}{2}\mathrm{e}^{(\frac{-1-\sqrt{1-J_1}}{J_1})t}u(t) - \frac{K}{2}\mathrm{e}^{(\frac{-1+\sqrt{1-J_1}}{J_1})t}u(t)$. Determine the stability properties of this system for different values of J_1 in the range $0 < J_1 < +\infty$. Reason your answer.

   (iii) Suppose that the impulse response of the system is $h(t) = \mathrm{e}^{(\frac{-1-j}{2})t}u(t) - \mathrm{e}^{(\frac{-1+j}{2})t}u(t)$ for some values of the constants associated with the motor. Using convolution find the system response to $x(t) = 3u(t+1) + u(t-2)$ (the use of Laplace transforms is not allowed here.) Reason your answer.