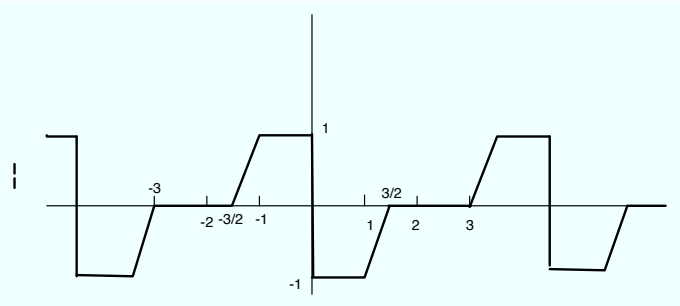
Signals and Systems - MAE 143A

Quiz 1 - Winter Quarter 2009

Student name and number _____

PLEASE READ:
You have 45 minutes. Please mark your papers with your name and surname, and student number. Read the questions very carefully. Answer the questions asked and do not second guess some presumed meaning. Marks are given for methods, concepts and expressions. Please show all your work so that we can see clearly that you know how to obtain the solution.

1. (8 points) Consider the following periodic signal $g(t)$.



(i) (2 points) Describe the signal from $-3 \leq t \leq 0$ using unit ramps and unit step functions.

(ii) (2 points) Describe now the signal from $0 \leq t \leq 3$.

(iii) (1 point) What is the fundamental period of the signal?

(iv) (2 points) Graph the signal $g(2t+2)$

(v) (1 point) By just looking at the symmetry of the graph, can you tell if the signal is even or odd?

Solution:

(i) From $-3 \leq t \leq 0$ the signal is described by

$$g(t) = 2\text{ramp}(t+\frac{3}{2}) - 2\text{ramp}(t+1), \quad -3 \leq t \leq 0\,.$$

(More than one solution is possible here)

(ii) We can obtain the expression for $g(t)$ in the range $0 \leq t \leq 3$ in several ways. The following are two of them:

(1) We can follow a similar approach to that of answer (i):

$$g(t) = -u(t) + 2\text{ramp}(t-1) - 2\text{ramp}(t-\frac{3}{2}), \quad 0 \leq t \leq 3\,.$$

(2) We can observe that the signal in the range $0 \leq t \leq 3$ is symmetric to the part in the range $-3 \leq t \leq 0$ about the origin. In other words, the signal is odd. We just have to rotate the expression we obtained in part (i) about the Y axis and then about the X axis. This corresponds to the following sequence of transformations: $g(t) \rightarrow g(-t)$ and $g(-t) \rightarrow -g(-t)$. In this way:

$$\begin{aligned} g(t) &= -(2\text{ramp}(-t + \frac{3}{2}) - 2\text{ramp}(-t + 1)) \\ &= -2\text{ramp}(-t + \frac{3}{2}) + 2\text{ramp}(-t + 1)\,. \end{aligned}$$

Both expressions from (1) and (2) are valid. These are not the only possibilities, other are valid as well. However, the purpose of this part (ii) (and that of part (i)) was to find an expression for the signal as a single equation (e.g. as in Homework 1, exercise 3)

(iii) The fundamental period of the signal can be obtained as $T = 3 - \frac{-3}{2} = \frac{9}{2}$.

(iv) To plot $g(2t + 2)$ we can apply the chain of transformations:

$$g(t) \overset{t \mapsto t+2}{\rightarrow} g(t + 2) \overset{t \mapsto 2t}{\rightarrow} g(2t + 2)$$

In other words, we get a sequence as in Figures 1 and 2.

(v) The signal $g(t)$ is odd because there is a symmetry about the origin: we can rotate the graph about the Y axis and then about the X axis and we would get the same graph.

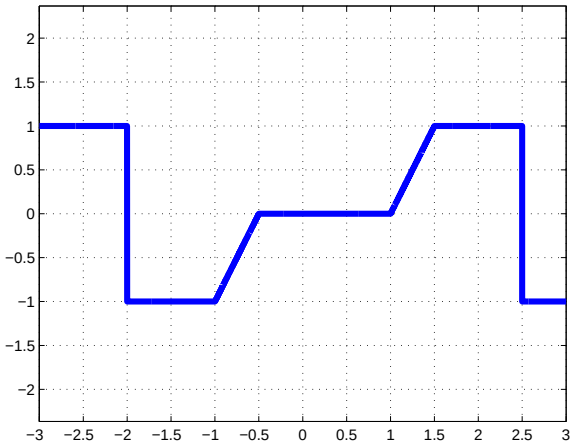


Figure 1: $g(t + 2)$

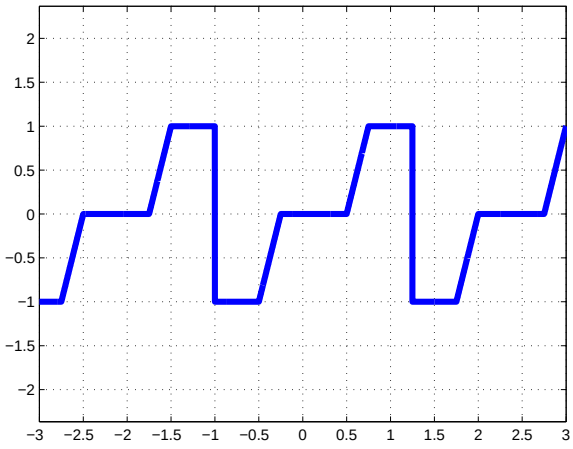


Figure 2: $g(2t + 2)$