MAE 143 A: Signals and Systems.
Homework #6

Assigned May 12. Due May 19

1. (a) A stable system has an ODE model given by

$$\dot{y}(t) + y(t) = \dot{x}(t)$$

Determine the magnitude and phase of its Frequency Response, $H(\omega)$. Based on an approximate plot of $|H(\omega)|$, classify the system into a low-pass, high-pass, or band-pass filter.

(b) Do the same as in (i) for a stable system with and ODE model given by:

$$\ddot{y}(t) + 4y(t) = x(t)$$

2. Suppose that the Frequency Response (FR) of a (stable) LTI system is given by the following function:

$$H(\omega) = \frac{j\omega}{4 + j\omega}$$

Determine:

(a) the output $y(t)$ when the input is $x(t) = \mathrm{e}^{j3t}$,

(b) the output $y(t)$ when the input is $x(t) = 8 + 2\sin(4t)$,

(c) the amplitude of the output $y(t)$ when the input is $x(t) = 5\cos(3t + \frac{\pi}{4})$,

(d) the output $y(t)$ when the input is $x(t) = \cos(2t - 10) + 2\sin(4t)$.

3. A periodic signal $x(t)$ with period $T = 2$ is specified as

$$x(t) = \mathrm{e}^{-\frac{t}{2}}, \qquad 0 \le t \le T.$$

Determine both the exponential and trigonometric Fourier series for $x(t)$.