MAE 143 A: Signals and Systems.
Homework #4.

Assigned Apr 21. Due Apr 28

1. Consider an approximation of the pendulum around the vertical upright position of $\theta = \pi$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{g}{L} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ML} \end{bmatrix} u$$

   Here, the constant values are $g = 10m/s^2$, $M = 0.025kg$, $L = 0.5m$. The variables are $\theta = x_1$ (angle the pendulum forms with the vertical) and $\dot{\theta} = x_2$ (rate of change of the angle variable.) Is it possible to control the angle and velocity of the linearized equation from any $x_1(0)$, $x_2(0)$ to any final x_1^f, x_2^f? Use the controllability check to verify it.

2. The following is a state-space model of a circuit which appeared in Homework 2 for some specific values of the circuit constants and after a matrix inversion operation. The input is a current i_S and the outputs are V and i_L (voltage at a capacitor and current in an inductor). Check that the system is not stable. However, check that the system is controllable.

$$\begin{bmatrix} \frac{dV}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ -1/3 & 0 \end{bmatrix} \begin{bmatrix} V \\ i_L \end{bmatrix} + \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix} i_S$$

   (*Observation:* This is a very interesting situation. Instability means that there are some current values i_S that are unsafe; i.e. they can burn out the circuit. However, when controllability is possible, there is a way to choose precise values of the input i_S to guarantee that the system behaves in a stable way. How to do this is explained in MAE 143B, linear control systems.)

3. Suppose that the impulse response of a physical system is approximately measured to be $h(t) = e^{-7t}\cos(2\pi t)u(t)$. Compute the response of a system to the signal $x(t) = 7u(t)$ using the convolution formula.