

# MAE 143 A: Signals and Systems.

## Homework #3.

Assigned Apr 14. Due Apr 21

1. A system is described by the following differential equation

$$7\frac{d^3y(t)}{dt^3} - 21\sin(y(t))\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 14\cos(y(t)) = U(t).$$

What is the state of the system? Rewrite the differential equation in the state-space form. Linearize the state-space equation about the point  $y = \frac{\pi}{2}, \dot{y} = 0, \ddot{y} = 0$  (and  $U(t) = 0$ ).

2. For different systems, we observe the following input-output experiments:
  - (a) System 1:  $x(t) \rightarrow \cos^2(2\pi f_0 t)$  and  $2x(t) \rightarrow 1 + \cos(4\pi f_0 t)$ , for some constant  $f_0$ . Is the system homogeneous? Why or why not? (*Hint*: Use trigonometric formulas to relate  $\cos(4\pi f_0 t)$  and  $\cos^2(2\pi f_0 t)$ )
  - (b) System 2:  $x(t) \rightarrow \cos(2\pi t)$  and  $x(t - \frac{\pi}{2}) \rightarrow \sin(2\pi t)$ . Is the system time-invariant? Why or why not? (*Hint*: Use trigonometric formulas)
  - (c) System 3:  $x(t) \rightarrow \delta(2t)$  and  $2x(t) \rightarrow \delta(t)$ . Is the system homogeneous? Why or why not?
3. An amplitude-modulator system takes in an audio signal  $x(t)$  and returns the output signal

$$y(t) = \cos(2\pi\omega_c t)x(t).$$

Classify the system as to homogeneity, time-invariance, additivity, LTI, and memory.

4. A system has a response that is the cube of its excitation. That is, to a signal  $x(t)$ , the system responds with the output  $y(t) = x(t)^3$ . Classify the system as to homogeneous, time-invariance, additivity, LTI and memory.