

$$\begin{array}{ll}
1 \xrightarrow{\mathcal{F}} \delta(f) & 1 \xrightarrow{\mathcal{F}} 2\pi\delta(\omega) \\
\delta(t) \xrightarrow{\mathcal{F}} 1 & \delta(t) \xrightarrow{\mathcal{F}} 1 \\
u(t) \xrightarrow{\mathcal{F}} (1/2)\delta(f) + 1/j2\pi f & u(t) \xrightarrow{\mathcal{F}} \pi\delta(\omega) + 1/j\omega \\
\text{sgn}(t) \xrightarrow{\mathcal{F}} 1/j\pi f & \text{sgn}(t) \xrightarrow{\mathcal{F}} 2/j\omega \\
\text{rect}(t) \xrightarrow{\mathcal{F}} \text{sinc}(f) & \text{rect}(t) \xrightarrow{\mathcal{F}} \text{sinc}(\omega/2\pi) \\
\text{sinc}(t) \xrightarrow{\mathcal{F}} \text{rect}(f) & \text{sinc}(t) \xrightarrow{\mathcal{F}} \text{rect}(\omega/2\pi) \\
\text{tri}(t) \xrightarrow{\mathcal{F}} \text{sinc}^2(f) & \text{tri}(t) \xrightarrow{\mathcal{F}} \text{sinc}^2(\omega/2\pi) \\
\text{sinc}^2(t) \xrightarrow{\mathcal{F}} \text{tri}(f) & \text{sinc}^2(t) \xrightarrow{\mathcal{F}} \text{tri}(\omega/2\pi) \\
\delta_{T_0}(t) \xrightarrow{\mathcal{F}} f_0\delta_{f_0}(f) & \delta_{T_0}(t) \xrightarrow{\mathcal{F}} \omega_0\delta_{\omega_0}(\omega) \\
e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} \delta(f-f_0) & e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega-\omega_0)
\end{array}$$

$$\cos(2\pi f_0 t) \xrightarrow{\mathcal{F}} (1/2)[\delta(f-f_0) + \delta(f+f_0)] \quad \cos(\omega_0 t) \xrightarrow{\mathcal{F}} \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

$$\sin(2\pi f_0 t) \xrightarrow{\mathcal{F}} (j/2)[\delta(f+f_0) - \delta(f-f_0)] \quad \sin(\omega_0 t) \xrightarrow{\mathcal{F}} j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$$

$$e^{-at} u(t) \xrightarrow{\mathcal{F}} \frac{1}{a+j2\pi f}, \quad \text{Re}(a) > 0 \quad e^{-at} u(t) \xrightarrow{\mathcal{F}} \frac{1}{a+j\omega}, \quad \text{Re}(a) > 0$$

$$te^{-at} u(t) \xrightarrow{\mathcal{F}} \frac{1}{(a+j2\pi f)^2}, \quad \text{Re}(a) > 0 \quad te^{-at} u(t) \xrightarrow{\mathcal{F}} \frac{1}{(a+j\omega)^2}, \quad \text{Re}(a) > 0$$

$$e^{-a|t|} \xrightarrow{\mathcal{F}} \frac{2a}{a^2 + (2\pi f)^2}, \quad \text{Re}(a) > 0 \quad e^{-a|t|} \xrightarrow{\mathcal{F}} \frac{2a}{a^2 + \omega^2}, \quad \text{Re}(a) > 0$$

$$e^{-\pi t^2} \xrightarrow{\mathcal{F}} e^{-\pi f^2} \quad e^{-\pi t^2} \xrightarrow{\mathcal{F}} e^{-\omega^2/4\pi}$$

Note: In these pairs, for periodic time functions, the fundamental period is $T_0 = 1/f_0 = 2\pi/\omega_0$.

There is an illustrated table of CTFT pairs in appendix D.

USE OF TABLES AND PROPERTIES

In this section are some examples that will illustrate use of table 10.1 and appendix D and the properties to find the CTFTs of some signals.

EXAMPLE 10.10

CTFT of Some Transformations of a Sine

If $x(t) = 10\sin(t)$, then find (a) the CTFT of $x(t)$, (b) the CTFT of $x(t-2)$, (c) the CTFT of $x(2(t-1))$, and (d) the CTFT of $x(2t-1)$.

■ Solution

(a) Using the linearity property and looking up the transform of the general sine form,

$$\sin(\omega_0 t) \xrightarrow{\mathcal{F}} j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$\sin(t) \xrightarrow{\mathcal{F}} j\pi[\delta(\omega + 1) - \delta(\omega - 1)]$$